2021 Japan-Taiwan Joint Online Workshop on Numerical Analysis and Inverse Problems

Date : 12 March (Fri), 2021
10:20–17:30 (JST), 9:20–16:30 (UTC+8)

Venue : Online by Zoom
The following venue is also available:
Room 203 (Applied Analysis Seminar Room)
Integrated Research Bldg. No.12, Kyoto University

Program (JST) (Subtract 1 hour to see Taiwan time)

9:50	Zoom Meeting Available (Speakers can check your screen sharing setting.)
10:20–10:30	Opening Address J-N. Wang and Y. Iso
10:30–11:00	Hau-Tieng Wu (Duke University) Some recent developments in nonstationary signal processing and associated biomedical applications
11:00–11:30	Michael Koch (Kyoto University) Fixed bounding domain based faster HMC inversion for simultaneous identification of spatial and interface properties of a seepage zone
11:30–12:00	Yi-Hsuan Lin (National Chiao-Tung University) Inverse problems for semilinear elliptic equations
12:00–12:30	Yu-Hsun Lee (Kyoto University) Numerical reliability of vortex filament evolution under the Biot-Savart law
	Lunch Break
14:00–14:30	Masahiro Yamamoto (The University of Tokyo) Mathematical analysis for inverse problems for fractional partial differential equations
14:30–15:00	Chin-Tien Wu (National Chiao-Tung University) A Weakly Kernel-Supervised Neural Network for Image Blind Deconvolution
15:00–15:30	Takaaki Nishida (Kyoto University) Routes to chaos in Rayleigh-Benard heat convection
15:45–16:15	Pengwen Chen (National Chung-Hsing University) Arnoldi Algorithms with Structured Orthogonalization
16:15–16:45	Kazunori Matsui (Kanazawa University) A projection method with boundary conditions involving the total pressure

16:45–17:15 Pu-Zhao Kow (National Taiwan University)
Reconstruction of an impenetrable obstacle in anisotropic inhomogeneous background

17:15–17:30 Closing Address
H. Fujiwara

Organizers Jenn-Nan Wang (National Taiwan University)
Yuusuke Iso (Kyoto University)
Hiroshi Fujiwara (Kyoto University)
Hitoshi Imai (Doshisha University)

Local Organizers Daisuke Kawagoe (Kyoto University)
Gi-Ren Liu (National Cheng Kung University)

Some recent developments in nonstationary signal processing and associated biomedical applications

Hau-Tieng Wu (Duke University)

Abstract

Compared with the snapshot health record information, long-term and high-frequency physiological time series provides health information from the other dimension. To extract useful biorhythm features from these time series for clinical usage, we encounter several challenges — the time series is usually composed of multiple oscillatory components with complicated statistical features, like time varying amplitude, frequency and non-sinusoidal pattern, and the signal quality is often impaired by inevitable noise. I will discuss recent progress in dealing with this kind of time series by the nonlinear-type time-frequency analysis along with the recent statistical inference results. I will also demonstrate its application to extracting physiological status from the peripheral venous pressure signal during surgery, and how to apply it to determine the standardized nonlinear phase function for quantifying the cardiopulmonary coupling.

Fixed bounding domain based faster HMC inversion for simultaneous identification of spatial and interface properties of a seepage zone

Michael C. Koch[1,*], Kazunori Fujisawa[1] and Akira Murakami[1]

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Abstract: We consider the simultaneous identification of the interface of an anomaly/cavity (embedded in a domain Ω) along with the Gaussian spatial random field $k(\mathbf{z},\omega)$ defined at each point $\mathbf{z} \in \Omega$, from noisy observation data. Here, ω is an element of the sample space Θ, and (Θ, F, P) is a complete probability space. Computational solution of the problem necessitates the discretization of the random field, which is conveniently done through the K-term truncated Karhunen-Loève (KL) expansion,

$$k(\mathbf{z},\omega) \approx \bar{k}(\mathbf{z}) + \sum_{i=1}^{K} \sqrt{\lambda_i}\varphi_i(\mathbf{z})\ {}^{1}\theta_i(\omega),$$

where $\bar{k}(\mathbf{z})$ is the mean function and ${}^{1}\theta_i(\omega)$ are standard normal random variables. λ_i and $\varphi_i(\mathbf{z})$ are obtained from the solution of the Integral Eigen Value Problem (IEVP)

$$\int_{\Omega} C(\mathbf{z},\mathbf{z}')\varphi_i(\mathbf{z}')\mathrm{d}\mathbf{z}' = \lambda_i\varphi_i(\mathbf{z}),$$

where $C(\mathbf{z},\mathbf{z}')$ is the autocovariance function, which is symmetric and positives semi-definite. The eigen values λ_i decay rapidly and for most practical purposes, the KL expansion involves a summation over a few terms only. Except for a few simple problems, the IEVP has to be solved numerically over Ω.

Statistical inversion is carried out in the Bayesian framework through a modern gradient based Markov Chain Monte Carlo (MCMC) algorithm called Hamiltonian Monte Carlo or HMC that avoids random walks by making gradient guided proposals. The parameter space is composed of two kinds of parameters ${}^{1}\boldsymbol{\theta}$ (representing the spatial field) and ${}^{2}\boldsymbol{\theta}$ (representing the interface). As the interface is updated at every step of HMC, the domain Ω is also updated and the IEVP has to be solved again. Computational complexity is further increased as the HMC gradients require the computation of the gradients $\frac{\partial \lambda_i}{\partial\ {}^{j}\boldsymbol{\theta}}$ and $\frac{\partial \varphi_i}{\partial\ {}^{j}\boldsymbol{\theta}}$, where $j = 1,2$.

For efficient computation, we leverage the domain independence property of the KL expansion, which states that the first and second order moments of a random process generated by the KL expansion are invariant to a change in the physical domain. Hence, λ_i and $\varphi_i(\mathbf{z})$ are computed only once using the highly efficient Nyström method on a rectangular bounding domain Ω_b, such that every realization in HMC of $\Omega \subseteq \Omega_b$. The eigen vectors are then interpolated to the FE mesh Gauss points at every step of HMC. The domain independence property further enables computational savings as the gradients of the eigen values and eigen vectors now no longer have to be calculated. This method is employed to determine the hydraulic conductivity $k(\mathbf{z},\omega)$ and the length and width of a piping zone embedded in a seepage zone shown in Fig 1, using experimental data.

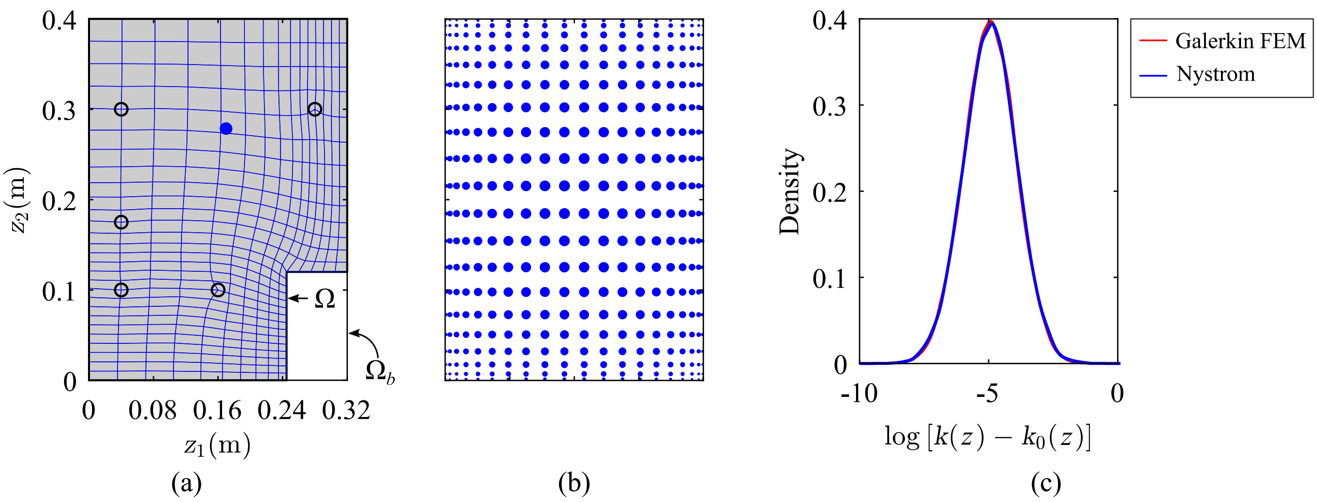

Fig. 1(a) Rectangular bounding domain Ω_b that bounds all HMC realizations Ω. Black circles represent hydraulic head observation points in the seepage zone. (b) 20×20 grid of Gauss-Legendre quadrature points to solve the IEVP using the Nyström method on Ω_b. (c) Domain independence of the KL expansion demonstrated through the probability density functions at the FEM Gauss point (marked as blue circle in (a)). Here, the IEVP is solved through Galerkin projection, on domain Ω and through the Nyström method on Ω_b.

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Inverse problems for semilinear elliptic equations

Yi-Hsuan Lin (National Chiao-Tung University)

Abstract

We introduce a method for solving Calderón type inverse problems for semilinear equations. The method is based on higher order linearizations, and it allows one to solve inverse problems for certain nonlinear equations in cases where the solution for a corresponding linear equation is not known.

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Numerical reliability of vortex filament evolution under the Biot-Savart law

Yu-Hsun Lee (Kyoto University)

Abstract

In this talk, we investigate the numerical reliability of self-induced motion of vortex filaments by Biot-Savart law which is represented as an integro-differential equation. We focus on the reconnection of vortex filaments which represents a singularity of fluid. The accelerated computation by GPU (Graphics Processing Unit) under the double precision environment enables us to estimate numerical properties with various parameters, and therefore we find a reliability criterion for this model. We also use quadruple precision arithmetic and 50 decimal digits arithmetic in order to evaluate accumulation of rounding errors in double precision arithmetic. We propose an algorithm to find the reconnection time by the use of an optimization.

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Mathematical analysis for inverse problems for fractional partial differential equations

Masahiro YAMAMOTO

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Abstract

We consider an initial boundary value problem for time-fractional diffusion-wave equation:

$$\begin{cases} \partial_t^\alpha u = -Au(x,t) + F(x,t), \\ u|_{\partial\Omega} = 0, \quad u(\cdot,0) = a, \quad \partial_t u(\cdot,0) = b \quad \text{if } 1 \le \alpha < 2, \end{cases}$$

where $x = (x_1, ..., x_d) \in \mathbb{R}^d$, and ∂_t^α, $0 < \alpha < 2$ denotes the Caputo derivative:

$$\partial_t^\alpha v(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-1-\alpha} \frac{d^n v(s)}{ds^n} ds$$

with $n-1 < \alpha < n$ with $n \in \mathbb{N}$ and the gamma function $\Gamma(n-\alpha)$, and $\Omega \subset \mathbb{R}^d$ is a smooth bounded domain, and $-A$ is a uniform elliptic operator of the second order whose coefficients may be time-dependent.

The conventional fractional derivative of order $\alpha < 1$ requires the first-order derivative and so we need some justification especially when we consider a weak solution. Another theoretical issue is the formulation of the initial condition because for $\alpha < \frac{1}{2}$ we cannot expect convenient continuity at $t = 0$.

First, in order to solving these issues, we define ∂_t^α in the subspaces $H_\alpha(0,T)$ in usual Sobolev-Slobodeckij spaces $H^\alpha(0,T)$, which is the clousure operator of the Caputo derivative

in $\{w \in C^1[0,T];\, w(0)=0\}$. Then we establish fundamental properties such as isomorphism between $\|\partial_t^\alpha w\|_{L^2(0,T)}$ and $\|w\|_{H_\alpha(0,T)}$, and we prove the unique existence of the solution to the initial boundary value problem with regularity properties. As for the details, we refer to Kubica, Ryszewska and Yamamoto [1].

Second, based on such a theory, I present the following topics among recent theoretical results for several inverse problems:

- Asymptotic behavior of the solution as $t \to \infty$
- Backward problems in time
- Determination of orders α
- Inverse coefficient problems

Unlike the case $\alpha = 1$: the classical diffusion equation, the time- fractional diffusion-wave equations indicate weak smoothing property and slow diffusion, which we characterize through the above-mentioned first and second topics.

Especially I would like to stress that fractional differential equations are promissing to the young, because this field is strongly supported by many real-world problems and applications such as anomalous diffusion of contaminants, and reliable mathematical researches are highly demanded.

References

[1] A. Kubica, K. Ryszewska, and M. Yamamoto, *Time-fractional Differential Equations: a Theoretical Introduction*, Springer Japan, Tokyo, 2020.

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A weakly kernel-supervised neural network for image blind deconvolution

Chin-Tien Wu (National Chiao-Tung University)

Abstract

Blind deconvolution (BD) is an important issue in image processing. The BD process generally involves two steps: kernel estimation and image deconvolution. To obtain a satisfactory recovered image using BD, many parameters tuning are required. Each set of parameters result in a different blur kernel and recovered image. There is no optimal parameters in general, due to variation of image priors. Parameters tuning is the state of the art for the BD to success. In this talk, we shall review some of the well known Blind deconvolution (BD) methods such as Levin's MAP x,k , Fergus's Variational Bayes mean field, Bronstein's Optimal sparse representation and Jiaya's two phase estimation based on L0 sparsity of the kernel, etc. We propose a kernel-supervised cycle-consistent adversarial network (KS-CCAN) for BD where the kernel supervisor is obtained from the kernel estimation of traditional BD methods. The KS-CCAN can be considered as an auto-tuning network for BD. Our preliminary results show that KS-CCAN is robust for various blur images.

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Routes to Chaos in Rayleigh-Bénard Heat Convection

by

Takaaki Nishida (Kyoto University) and Chun-Hsiung Hsia (National Taiwan University)

Roll-type Solutions in the large.

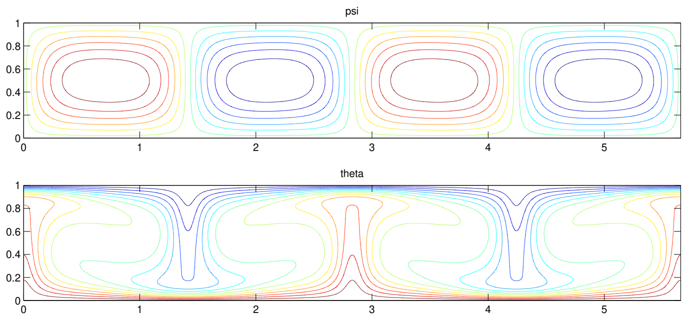
In order to obtain the roll-type solution of Rayleigh-Bénard heat convection for large Rayleigh number we use the stream function and temperature as unknowns :

$$\frac{\partial}{\partial t}\Delta\psi = \mathcal{P}_r\Delta^2\psi + \mathcal{P}_r\mathcal{R}_a\frac{\partial\theta}{\partial x} + \frac{\partial\psi}{\partial z}\frac{\partial\Delta\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial z} \tag{1}$$

$$\frac{\partial\theta}{\partial t} = \Delta\theta + \frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}\ . \tag{2}$$

Since the boundary conditions are stress free, the solution has the Fourier series expansion. We express (1)(2) with Fourier series expansions and use Galerkin method and Newton method for the solutions of the stationary system of (1)(2).

$\mathcal{R}_a = 20.0\times\mathcal{R}_c\ ,\ \mathcal{P}_r = 10.0$



The Hopf bifurcation from the stationary roll-type solution of mode (2,1)(4,1).

N	$\mathcal{R}_a\,/\,\pi^4$	$r\ =\ \mathcal{R}_a\,/\,\mathcal{R}_c$	λ
32	$277.76301\cdots$	$41.15007\cdots$	$i\times 40.75766\cdots$
40	$277.40075\cdots$	$41.09640\cdots$	$i\times 40.72751\cdots$
48	$277.36731\cdots$	$41.09145\cdots$	$i\times 40.72462\cdots$
56	$277.36495\cdots$	$41.09110\cdots$	$i\times 40.72441\cdots$
64	$277.36481\cdots$	$41.09108\cdots$	$i\times 40.72440\cdots$

The time periodic and chaotic solutions can be obtained by the numerical computations of time evolution of the system (1)(2) with cut-off Fourier series expansions after time discretization.

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Arnoldi algorithms with structured orthogonalization

Pengwen Chen (National Chung-Hsing University)

Abstract

We study a stability preserved Arnoldi algorithm for matrix exponential in the time domain simulation of large-scale power delivery networks (PDNs), which are formulated as semi-explicit differential algebraic equations (DAEs). The solution can be decomposed to a sum of two projections, one in the range of the system operator and the other in its null space. The range projection can be computed with a shift-and-invert Krylov subspace method. The other projection can be computed with the algebraic equations. Differing from the ordinary Arnoldi method, the orthogonality in the Krylov subspace is replaced with the semi-inner product induced by the positive semidefinite system operator. With proper adjustment, numerical ranges of the Krylov operator lie in the right half-plane, and we obtain theoretical convergence analysis for the modified Arnoldi algorithm in computing phi-functions. Last, simulations on RLC networks are demonstrated to validate the effectiveness of the Arnoldi algorithm.

A projection method with boundary conditions involving the total pressure

Kazunori Matsui

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Let $T > 0$ and let Ω be a bounded Lipschitz domain in $\mathbb{R}^d$ $(d = 2, 3)$ with the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$ $(\Gamma_1 \cap \Gamma_2 = \emptyset)$. We consider the following Navier–Stokes problem: Find two functions $u : \Omega \times [0, T] \to \mathbb{R}^d$ and $p : \Omega \times [0, T] \to \mathbb{R}$ such that

$$\begin{cases} \dfrac{\partial u}{\partial t} + D(u, u) - \nu\Delta u + \dfrac{1}{\rho}\nabla P = f & \text{in } \Omega \times (0, T), \\ \operatorname{div} u = 0 & \text{in } \Omega \times (0, T), \\ u = 0 & \text{on } \Gamma_1 \times (0, T), \\ u \times n = 0 & \text{on } \Gamma_2 \times (0, T), \\ P = p^b & \text{on } \Gamma_2 \times (0, T), \\ u(0) = u_0 & \text{in } \Omega, \end{cases} \tag{NS}$$

where $\nu, \rho > 0$, $D(v, w) := (\nabla \times v) \times w$, $P := p + \frac{\rho}{2}|u|^2$, $f : \Omega \times (0, T) \to \mathbb{R}^d$, $p^b : \Gamma_2 \times (0, T) \to \mathbb{R}$, $u_0 : \Omega \to \mathbb{R}^d$, n is the unit outward normal vector for Γ, and "$\times$" is the cross product in $\mathbb{R}^d$. If $d = 2$, then we define $v \times w := v_x w_y - v_y w_x$ and $(\nabla \times v) \times w := (w_y(\partial_y v_x - \partial_x v_y), w_x(\partial_x v_y - \partial_y v_x))$ for all $v = (v_x, v_y), w = (w_x, w_y)$. In the first equation of (NS), we have used

$$D(u, u) + \frac{1}{\rho}\nabla P = (u \cdot \nabla)u + \frac{1}{\rho}\nabla p.$$

On Γ_2, we assume the boundary condition including the pressure value $p + \frac{\rho}{2}|u|^2 (= P)$, which is called a total pressure or a stagnation pressure. Although both the measured values of the total and usual pressure by a Pitot tube are dependent on the yaw angle of the Pitot tube, it is known that the effect on the total pressure is smaller than the effect on the usual pressure.

In this talk, we propose a time-discretized problem for (NS) by using an idea of the projection method. Let $\tau (:= T/N < 1, N \in \mathbb{N})$ be a time increment and let $t_k := k\tau$ $(k = 0, 1, \ldots, N)$. We set $u_0^* := u_0$ and calculate u_k^*, u_k, P_k $(k = 1, 2, \ldots, N)$ by repeatedly solving the following problems (Step 1) and (Step 2).

(Step 1) Find $u_k^* : \Omega \to \mathbb{R}^d$;

$$\begin{cases} \dfrac{u_k^* - u_{k-1}}{\tau} + D(u_{k-1}^*, u_k^*) - \nu\Delta u_k^* = f(t_k) & \text{in } \Omega, \\ u_k^* = 0 & \text{on } \Gamma_1, \\ u_k^* \times n = 0 & \text{on } \Gamma_2, \\ \operatorname{div} u_k^* = 0 & \text{on } \Gamma_2. \end{cases}$$

(Step 2) Find $P_k :\to \mathbb{R}, u_k :\to \mathbb{R}^d$;

$$\begin{cases} -(\tau/\rho)\Delta P_k = -\operatorname{div} u_k^* & \text{in } \Omega, \\ \dfrac{\partial P_k}{\partial n} = 0 & \text{on } \Gamma_1, \\ P_k = p^b(t_k) & \text{on } \Gamma_2, \end{cases}$$

$$u_k = u_k^* - (\tau/\rho)\nabla P_k \quad \text{in } \Omega.$$

We show a stability of the scheme and establish error estimates in suitable norms between the solutions to the scheme and (NS).

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Reconstruction of an impenetrable obstacle in anisotropic inhomogeneous background

Pu-Zhao Kow (National Taiwan University)

Abstract

In this talk, we consider the problem of determining the shape of obstacles in the acoustic wave scattering with an anisotropic inhomogeneous medium. Precisely, we can reconstruct the obstacle by the eigenvalues and eigenfunctions of the far-field operator. We also provide some numerical simulations.