2022 Japan-Taiwan Joint Online Workshop on Numerical Analysis and Inverse Problems

Date : 25 March (Fri), 2022
10:50–16:40 (JST), 9:50–15:40 (UTC+8)
Venue : Online by Zoom
The following venue is also available:
Room 203 (Applied Analysis Seminar Room)
Integrated Research Bldg. No.12, Kyoto University

Program in JST; Subtract 1 Hour to Taiwan Time

10:30	Zoom Meeting Available (Speakers can check your device.)
10:50–11:00	Opening Address J-N. Wang and Y. Iso
11:00–11:40	Yi-Hsuan Lin (National Yang Ming Chiao Tung University) Inverse problems for fractional equations with a minimal number of measurements
11:40–12:20	Masaki Imagawa (Kyoto University) On convergence estimation of elliptic regularization applied to boundary value problems of advection equations
12:20–13:00	Masato Kimura (Kanazawa University) Particle dynamics in bounded domains
	Lunch Break
14:00–14:40	Takaaki Nishida (Kyoto University) An example of thermal convection with non-uniform heat supply
14:40	Self Introduction of Participants
15:10–15:50	Chun-Hsiang Tsou (National Central University) Inverse scattering by corners and regular transmission eigenfunctions
15:50–16:30	Pu-Zhao Kow (University of Jyväskylä)) Non-scattering phenomena of quadrature domains
16:30	Closing Address H Imai, H. Fujiwara

Organizers Jenn-Nan Wang (National Taiwan University)
Yuusuke Iso (Kyoto University)
Hitoshi Imai (Doshisha University)
Hiroshi Fujiwara (Kyoto University)

1. Yi-Hsuan Lin
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Title: Inverse problems for fractional equations with a minimal number of measurements

Abstract:

We study several inverse problems associated with a fractional differential equation of the following form:

$$(-\Delta)^s u(x) + \sum_{k=0}^{N} a^{(k)}(x)[u(x)]^k = 0, \quad 0 < s < 1, \ N \in \mathbb{N} \cup \{0\} \cup \{\infty\},$$

which is given in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 1$. For any finite N, we show that $a^{(k)}(x)$, $k = 0, 1, \ldots, N$, can be uniquely determined by $N + 1$ different pairs of Cauchy data in $\Omega_e := \mathbb{R}^n \backslash \overline{\Omega}$. If $N = \infty$, the uniqueness result is established by using infinitely many pairs of Cauchy data. The results are highly intriguing in that it generally does not hold true in the local case, namely $s = 1$, even for the simplest case when $N = 0$, a fortiori $N \geq 1$. The nonlocality plays a key role in establishing the uniqueness result. We also establish several other unique determination results by making use of a minimal number of measurements. Moreover, in the process we derive a novel comparison principle for nonlinear fractional differential equations as a significant byproduct.

2. Masaki Imagawa
Graduate School of Informatics, Kyoto University, Japan
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Title: On convergence estimation of elliptic regularization applied to boundary value problems of advection equations

Abstract: We consider boundary value problems of an advection equation on a bounded Lipschitz domain with a piecewise C^1 boundary, and we pose homogeneous inflow boundary condition to the equation. Aiming at stable numerical computation, we introduce an elliptic regularization term, which is called a viscosity term, to the equation, and we give convergence estimates of regularized solutions to the original solution with respect to a viscous parameter. We also show some numerical results.

References

[1] C. Bardos, Problèmes aux limites pour les équations aux dérivées partielles du premier ordre à coefficients réels ; théorèmes d'approximation; application à l'équation de transport, *Ann. Sci. École Norm. Sup.(4)*, **3** (1970), pp. 185–233.

[2] D. A. Di Pietro and A. Ern, *Mathematical aspects of discontinuous Galerkin methods*, Springer(2012).

[3] G.-M. Gie, M. Hamouda, C.-Y. Jung and R. Temam, *Singular perturbations and boundary layers*, Springer, Chem(2018).

[4] C. Johnson and U. Nävert, An analysis of some finite element methods for advection-diffusion problems, in *Analytical and Numerical Approaches to Asymptotic Problems in Analysis - Proceedings of the Conference on Analytical and Numerical Approaches to Asymptotic Problems*, North-Holland Mathematics Studies **47** (1981), pp. 99–116.

3. Masato Kimura
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Title: Particle dynamics in bounded domains

Abstract: We consider the dynamics of interacting particle systems where particles are confined to a bounded domain. To control the particles not to escape the domain, there are many boundary rules to the particle on the boundary. But under these boundary rules, a rigorous solution concept for particle trajectory is missing. Our aim is therefore to establish a satisfactory analytical framework for particle dynamics where particles are confined to a bounded, possibly nonconvex domain. We consider two problems in this talk, one is a first-order ODE system, when particles hit the boundary, we consider an instant change in velocity. The other one is a second-order ODE system, we consider collisions with the boundary are described as purely elastic collisions. This provides a theoretical basis for the boundary conditions of particle methods. This is joint work with Zhenxing Yang and Patrick van Meurs, Kanazawa University.

References

[1] M. Kimura, P. van Meurs, Z. X. Yang: Particle dynamics subject to impenetrable boundaries: existence and uniqueness of mild solutions. SIAM J. Math. Anal. Vol. 51, No. 6 (2019) pp. 5049–5076. (arXiv:1812.08969)

[2] M. Kimura, P. van Meurs, and Z. X. Yang: Particle dynamics with elastic collision at the boundary: existence and partial uniqueness of solutions. Acta Applicandae Mathematicae, Vol. 147, No. 1 (2021) 1–26. (arXiv:2011.14485)

4. Takaaki Nishida
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Title: An example of thermal convection with non-uniform heat supply

Abstract: Stommel (1950) considered a model of thermal convection in horizontally long liquid layer with non-uniform heat supply under gravity. He obtained approximate solutions by asymptotic expansion of the equations with respect to a dimension-less parameter and showed its picture of contour line of the stream function and isothermal line. It may be considered as a simplest model of thermal effect to the ocean current. Here we show the existence of stationary solutions under some assumptions *depth* $\ll$ *horizontal length* and under stress free boundary conditions on the velocity. Also we show pictures of the isothermal line and the contour lines of solutions.

5. Chun-Hsiang Tsou
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Title: Inverse scattering by corners and regular transmission eigenfunctions

Abstract: In this talk, I will present my recent work on the inverse scattering problems. In this work, we tried to recover the polygonal scatters, which are characterized by the piecewise constant coefficient in the principal part of the Helmholtz equation. The main results are twofold, the stability estimation and the regularity of the transmission eigenfunctions. We have obtained a logarithmic stability estimation for the polygonal inclusions in terms of the far-field patterns. Moreover, an Hölder type regularity of the transmission eigenfunctions is also deduced. This is the joint work with H. Liu at City University, Hong Kong.

6. Pu-Zhao Kow
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Title: Non-scattering phenomena of quadrature domains

Abstract: In this talk, we are focusing on non-scattering obstacles. Precisely, we consider the following inverse scattering problem: Given a penetrable obstacle, does there exist an incident wave that does not scatter? In fact, each quadrature domain is non-scattering. There exists quadrature domain with inward cusp points as well as double points. We will also exhibit the procedure of constructing quadrature domains using an obstacle problem (i.e. partial balayage), which involving variational principle.