2024 Japan-Taiwan Joint Workshop on Numerical Analysis and Inverse Problems

Date: 9 (Sat.), 10 (Sun.), March, 2024

Venue: Room 111 (総合講義室 2) at Faculty of Engineering

Integrated Research Bldg. Kyoto University.

Program

9 (Sat.), March

10:50-11:00 Opening Address Jenn-Nan Wang and Hiroshi Fujiwara

(Chair : Masato Kimura; Kanazawa University)

11:00-11:40 Jenn-Nan Wang (王 振男)

Bayesian approach to the inverse medium scattering problem

11:40-12:20 Hiroshi Fujiwara (藤原 宏志)

Numerical feasibility of optical tomography by the stationary radiative transport equation

(Chair : Satoshi Taguchi; Kyoto University)

14:00-14:40 Masato Kimura (木村 正人)

Energy dissipation structure of the Zener-type viscoelastic wave equation and its applications

14:40-15:20 Shih-Hsien Yu (尤 釋賢)

Navier-Stokes equations in gas dynamics: Greens function, singularity, and well-posedness

(Chair : Jenn-Nan Wang; National Taiwan University)

15:40-16:20 I-Kun Chen (陳 逸昆)

Geometric effects on $W^{1,p}$ regularity of the stationary linearized Boltzmann equation

16:20-17:00 Tomoyuki Miyaji (宮路 智行)

Traveling waves and the Neimark--Sacker bifurcation with discrete rotational symmetry

17:00-18:00Discussion

10 (Sun.), March

(Chair : Daisuke Kawagoe; Kyoto University)

10:50-11:10 Masaki Imagawa (今川 真城)

On a boundary value problem of a first-order PDE and its regularized problems

11:10-11:30 Kousei Takahashi (高橋 幸聖)
Numerical computation for transonic potential flow with particular solutions of Chaplygin's equation

11:30-11:50 Takuma Tomita (富田卓磨)
Numerical analysis of a rarefied gas flow past a circular disk: peculiar behavior near the sharp edge

11:50-12:00 Group Photo

(Chair : Hiroshi Fujiwara; Kyoto University)
13:30-14:10 Hiroshi Isakari (飯盛 浩司)
Towards a topology optimisation of wave devices with wide working bandwidth

14:10-14:50 Yu-Chen Shu (舒 宇宸)
Numerical investigations to time-fractional Allen-Cahn equation

(Chair : I-Kun Chen; National Taiwan University)
15:10-15:50 Pu-Zhao Kow (邱 普照)
Some free boundary methods and their application to inverse scattering problems

15:50-16:30 Hiroshi Takase (髙瀬 裕志)
Inverse source problems for wave equations with inverse square potentials

16:30-16:40 Closing Address Yuusuke Iso

Organizers

Jenn-Nan Wang (National Taiwan University) 王 振男 (臺灣大學)
Yuusuke Iso (Kyoto University) 磯 祐介 (京都大学)
Hitoshi Imai (Doshisha University) 今井 仁司 (同志社大学)
Hiroshi Fujiwara (Kyoto University) 藤原 宏志 (京都大学)

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Bayesian approach to the inverse medium scattering problem

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Abstract

In this talk, I plan to discuss the inverse scattering problem of determining an unknown refractive index from the far-field measurements using the Bayesian approach. Our aim is to prove the Bernstein-von Mises theorem for this inverse scattering problem when the unknown coefficient is in a class of piecewise constant functions. We also provide numerical simulations based on the MCMC algorithm combining with a Gibbs-type sampling and MALA to verify the Bernstein-von Mises theorem. This talk is based on joint works with Takashi Furuya.

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Numerical Feasibility of Optical Tomography by the Stationary Radiative Transport Equation

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Abstract

We study reconstruction of optical properties of biological tissue by using near infrared light which is scattered and absorbed during propagation. In this talk we use the stationary radiative transport equation (RTE) as a mathematical model for NIR-light propagation, and show numerical reconstruction of attenuated coefficients in RTE from simulated data. The discontinuity of the solution to RTE induced by the boundary condition plays an important role in the proposed approach. We also introduce a numerical scheme for the boundary value problem of RTE by the piecewise constant approximation in order to detect discontinuities of the solution efficiently. This talk is based on a collaboration with I.-K. Chen (National Taiwan University), D. Kawagoe (Kyoto University), and N. Oishi (Kyoto University).

References

[1] I.-K. Chen, H. Fujiwara, and D. Kawagoe, Tomography from scattered signals obeying the stationary radiative transport equation, *Mathematics for Industry* **37** (2023) pp.27–46

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Energy dissipation structure of the Zener-type viscoelastic wave equation and its applications

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Abstract

We consider a Zener-type viscoelastic wave equation in a bounded domain, which consists from n Maxwell-type viscoelastic components connected in parallel and a dumping term. Based on the observation that a physically natural energy dissipation equality holds for a smooth solution, we develop the following mathematical analysis for the Zener-type viscoelastic wave equation: 1) A time discretized model is constructed and its energy dissipation equality is proved. 2) Existence of a weak solution is shown by a compactness argument for the time discretized solution as the time increment tends to zero. 3) The energy dissipation equality is shown for weak solutions and the uniqueness of a weak solution follows from it. 4) The exponential decay property as time tends to infinity is proved even for the case without the dumping term.

This research is an extension of previous studies[1, 2, 3], and is a joint work with J.-T. Li (Kanazawa U.), Y. Liu (Kyoto U.), G. Nakamura (Hokkaido U.), H. Notsu (Kanazawa U.), and Y. Ueda (Kobe U.).

References

[1] M. Kimura, H. Notsu, Y. Tanaka, and H. Yamamoto: The gradient flow structure of an extended Maxwell viscoelastic model and a structure-preserving finite element scheme. Journal of Scientific Computing, Vol.78 (2019) pp.1111-1131. doi:10.1007/s10915-018-0799-2

[2] J.-T. Li: Mathematical analysis of the Zener-type viscoelastic wave equation. Division of Mathematical and Physical Science, Graduate School of Natural Science and Technology, Kanazawa University, Master Thesis (2023 February).

[3] H. Yamamoto: Energy gradient flow structure of the Zener viscoelastic model and its finite element analysis. Master theis, Division of Mathematical and Physical Sciences, Graduate School of Natural Science and Technology, Kanazawa University (February 2019) 36 pages (in Japanese).

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Navier-Stokes Equations in Gas Dynamics: Greens Function, Singularity, and Well-Posedness

Tai-Ping Liu [1,2] and Shih-Hsien Yu [2]

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Abstract

We are interested in the well-posedness theory and the propagation of singularity in the weak solutions for the initial value problem. Our approach is to convert the differential equations into integral equations on the level of weak solutions. This depends on exact analysis of the associated linear equations and their Green's functions. We carry out our approach for the Navier-Stokes equations in gas dynamics. Local in time as well as time- asymptotic behaviors of weak solutions, and the continuous dependence of the solutions on their initial data are established.

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Geometric effects on $W^{1,p}$ regularity of the stationary linearized Boltzmann equation

I-Kun Chen[2], Chun-Hsiung Hsia[2], Daisuke Kawagoe[1], Jhe-Kuan Su [2]

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Abstract

We study the incoming boundary value problem for the stationary linearized Boltzmann equation in bounded convex domains. The geometry of the domain has a dramatic effect on the space of solutions. We prove the existence of solutions in $W^{1,p}$ spaces for $1 \leq p < 2$ for small domains. In contrast, if we further assume the positivity of the Gaussian curvature on the boundary, we prove the existence of solutions in $W^{1,p}$ spaces for $1 \leq p < 3$ provided that the diameter of the domain is small enough. In both cases, we provide counterexamples in the hard sphere model; a bounded convex domain with a flat boundary for $p = 2$, and a small ball for $p = 3$.

References

[1] Geometric effects on $W^{1,p}$ regularity of the stationary linearized Boltzmann equation arXiv preprint arXiv:2311.12387, 2023

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Traveling waves and the Neimark–Sacker bifurcation with discrete rotational symmetry

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Abstract

The Neimark–Sucker bifurcation is a bifurcation of a fixed point in discrete-time dynamical systems, which leads to the appearance of a branch of invariant closed curves. We consider this bifurcation for smooth maps with $\mathbb{Z}_n$-symmetry and point out that a branch of traveling waves corresponding to invariant closed curves bifurcates. We discuss a method of numerical continuation of the bifurcating traveling waves. We also discuss its application to a system of delay-difference equations, a traffic flow model proposed by the authors.

References

[1] P. Chossat and M. Golubitsky, Iterates of maps with symmetry, *SIAM J. Math. Anal.* **19** (1988) 1259–1270.

[2] M. Golubitsky, I. Stewart, and D.G. Schaeffer, *Singularities and Groups in Bifurcation Theory II*, Springer, New York, 1988.

[3] Y.A. Kuznetsov, *Elements of Applied Bifurcation Theory*, Fourth edition Springer Nature, Switzerland, 2023.

[4] R. Mazrooei-Sebdani and Z. Eskandari, Neimark-Sacker Bifurcation with $\mathbb{Z}_n$-Symmetry and a Neural Application, *Qual. Theory Dyn. Syst.* **18** (2019) 931–946.

[5] K. Okamoto, T. Miyaji, and A. Tomoeda, Nonlinear delay difference equation with bistability as a new traffic flow model, submitted.

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On a boundary value problem of a first-order PDE and its regularized problems

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Abstract

In the study of PDEs, it is often useful to consider approximate equations for the equations of interest. In this talk, we will consider a bounday value preblems for first-order PDE and the regularized elliptic problems that approximate it, which is obtained by adding the Laplacian with small parameters to the original equation. We introduce convergence estimates and numerical examples of approximate solutions. This talk is based on joint work with Dr. Daisuke Kawagoe (Kyoto Univ.).

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Numerical computation for transonic potential flow with particular solutions of Chaplygin's equation

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Abstract

We introduce a numerical method based on the hodograph transformation for the plane transonic potential flow past a wing. The plane transonic potential flow is governed by Chaplygin's equation in hodograph, which is a mixed type linear partial differential equation. However, hodograph transformation generally suffers from its non-univalency, e.g. limiting line singularity. We will overcome this difficulty by analytic continuation of its solutions on the corresponding Riemann surface.

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Numerical analysis of a rarefied gas flow past a circular disk: peculiar behavior near the sharp edge

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Abstract

In the framework of rarefied gas dynamics, a flow around a body induces a non-uniform temperature field, even if the body has a uniform temperature. This effect is called thermal polarization and one of the most significant differences from ordinary fluid dynamics, in which flow and temperature fields are determined independently. From this point of view, we study a uniform flow past a circular disk numerically. The boundary value problem is composed of the Bhatnagar-Gross-Krook model of the Boltzmann equation and the diffuse reflection boundary condition. To solve this BVP, we construct a numerical scheme based on the method of characteristics, and performed large-scale computing by using a supercomputer to obtain the detailed profiles of velocity distribution function. From the numerical result, it is revealed that thermal polarization is localized near the edge of the disk in the continuum limit. In the talk, after describing the method of numerical analysis, we present numerical results with emphasis on the peculiar behavior near the edge.

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Towards a topology optimisation of wave devices with wide working bandwidth

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Abstract

Topology optimisation [1], originally proposed in the field of mechanics aiming to achieve light and stiff structural members, is now expanding its application fields to various engineering fields such as heat, fluid, and wave device designs [2]. Usually, the existing topology optimisations for wave devices assume a "monochrome" incident excitation oscillating with a single frequency. As a result, the performance of the derived configuration can be sensitive to the variation in the excitation frequency. Since the frequency can deviate from the assumed value at the design stage in a practical situation, it is important to provide wave devices with robust performance against the frequency perturbation. In the presentation, we shall introduce our recent effort to realise such a robust topology optimisation. Our method [3, 4] rely on the fast frequency sweep based on the boundary integral method [5], and the level-set method utilising the topological derivative [6].

References

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[2] H Isakari K Kuriyama S Harada T Yamada T Takahashi and T Matsumoto 2014 A topology optimisation for three-dimensional acoustics with the level set method and the fast multipole boundary element method *Mechanical Engineering Journal* **1**(4) CM0039

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[6] S Amstutz and H Andrä 2006 A new algorithm for topology optimization using a level-set method. *Journal of Computational Physics* **216**(2) 573-588

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Numerical Investigations to Time-Fractional Allen-Cahn Equation

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Abstract

In this talk, we present a dissipative functional for the time-fractional Allen-Cahn equation and examine the properties analytically and numerically. The time differential operator here is realized by a Caputo-type differential operator. In order to answer the open problem, the dissipative property of classical energy, the dissipative functional is derived from the formulation of classical energy and Caputo differential operator. We show the functional is unconditional dissipation, and the decay rate is shown to follow $t^{\alpha-1}$. In addition, we provide the error analysis of the numerical solution. The analysis depends on the regularity of the solution and the implicit-explicit notion inherited in the convex splitting method. We show the convergence order is of t^{α} regardless of the chosen convex splitting method, and the numerical solutions obey the maximum principle.

References

[1] Q. Du, J. Yang, and Z. Zhou, 2020 Time-fractional allen–cahn equations: analysis and numerical methods, *Journal of Scientific Computing*, **85** , pp. 1–30

[2] T. Tang, H. Yu, and T. Zhou, 2019 On energy dissipation theory and numerical stability for time-fractional phase-field equations, *SIAM Journal on Scientific Computing*, **41**, pp. A3757–A3778

[3] H.-l. Liao, T. Tang, and T. Zhou, 2021 An energy stable and maximum bound preserving scheme with variable time steps for time fractional allen–cahn equation, *SIAM Journal on Scientific Computing*, **43**, pp. A3503–A3526.636

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Some free boundary methods and their application to inverse scattering problems

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Abstract

In this talk, we consider the inverse scattering problem of acoustic waves, which can be modeled by the Helmholtz equation. We will focus on the non-scattering phenomena. This topic has been presented via Zoom two years ago (`https://www-an.acs.i.kyoto-u.ac.jp/tj2022/`). Now I will elaborate this talk according to some of my recent works.

Some of the non-scattering domain can be constructed using partial balayage [1] or the minimizing problem (with free boundary) [2]. We also discuss the possible scattering and non-scattering behavior of an anisotropic and inhomogeneous Lipschitz medium, by connecting the anisotropic non-scattering problem to a Bernuolli type free boundary problem [3].

References

[1] P.-Z. Kow, S. Larson, M. Salo, and H. Shahgholian, *Quadrature and non-scattering domains for the helmholtz equation*, Potential Anal., 60 (2024), pp. 387–424. MR4696043, Zbl:7798456, doi:10.1007/s11118-022-10054-5. The results in the appendix are well-known, and the proofs can found at `arXiv:2204.13934`.

[2] P.-Z. Kow, M. Salo, and H. Shahgholian, *A minimization problem with free boundary and its application to inverse scattering problems*, arXiv preprint, (2023). `2303.12605`. To appear in Interfaces Free Bound.

[3] ——, *On scattering behavior of corner domains with anisotropic inhomogeneities*, arXiv preprint, (2023). `2309.11213`.

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Inverse source problems for wave equations with inverse square potentials

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Abstract

We will show the global Lipschitz stability estimate for an inverse source problem of the one-dimensional wave equation with inverse square potential with the singularity inside a domain. The key tool is the global Carleman estimate applicable to equations with singular potentials. Because of the singularity of the potentials, conventional perturbation methods for bounded potentials cannot be applied, and a new weight function must be chosen that resolves this difficulty. Also, the regularity of the solution expected from the general theory of well-posedness currently only allows us to prove this estimate in one dimension.

References

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