Modeling of slip on a fault during an earthquake: point-source approximation

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Some parts of the Earth's crust dynamically rupture and slip during earthquakes, and their deformation is often represented by the equation of motion of an elastic domain $\Omega(\ni \boldsymbol{x})$:

$$\rho\, \ddot{\boldsymbol{u}} = \nabla \cdot \left(\lambda\, (\nabla \cdot \boldsymbol{u})\, \boldsymbol{I} + \mu \left(\nabla \boldsymbol{u} + \left(\nabla \boldsymbol{u}\right)^{T}\right)\right) \tag{1}$$

where $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t)$ is the displacement vector, and $\rho = \rho(\boldsymbol{x})$, $\lambda = \lambda(\boldsymbol{x})$, and $\mu = \mu(\boldsymbol{x})$ are the density and Lamé parameters in the heterogeneous Earth, respectively. While the domain Ω is a continuum, the velocity during the coseismic slip must be discontinuous across the ruptured region $\Gamma(\subset \Omega)$, called a fault. Seismologists focus on the gap of displacement and velocity across Γ. Here, we assume that Γ is planar and lying along x_1-x_2-plane, and the gap is parallel to x_1-axis, for simplicity. Then, the slip at $\boldsymbol{s}(\in \Gamma)$ is defined as:

$$D(\boldsymbol{s}, t) := \lim_{\varepsilon \downarrow 0} \left[u_1(\boldsymbol{s} + \varepsilon \boldsymbol{\nu}, t) - u_1(\boldsymbol{s} - \varepsilon \boldsymbol{\nu}, t)\right], \tag{2}$$

where $\boldsymbol{\nu} = (0, 0, 1)^T$ is a normal vector to Γ.

The forward and inverse problems to model slip D and slip velocity $\dot{D}$ are a major topic in seismology. When we observe ground velocity $\dot{\boldsymbol{u}}$ at sufficiently far field, the spatial extent of Γ would be negligible, and $\dot{\boldsymbol{u}}$ is synthetically represented as follows:

$$\dot{\boldsymbol{u}}(\boldsymbol{x}, t) = \int_0^t \dot{D}(\tau)\, G(\boldsymbol{x}, t - \tau; \boldsymbol{s})\, d\tau, \tag{3}$$

where G is a propagator obtained synthetically or empirically. Some empirical laws are known for the moment and spectra of $\dot{D}$ in this point-source approximation. The author will discuss their main points and a stochastic model to represent the empirical laws.

References

[1] Hirano, S. (2022). Source time functions of earthquakes based on a stochastic differential equation, *Scientific Reports*, **12**:3936, `https://doi.org/10.1038/s41598-022-07873-2`