A numerical scheme for severely ill-posed nonlinear inverse problems with a regularized Moore-Penrose pseudoinverse

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Inverse problems appearing in optical tomography are nonlinear and severely ill-posed. Among non-iterative schemes, the Born and Rytov approximations are well known. These approximations, however, require linearization of the nonlinear inverse problem. Then the linearized inverse problem can be solved using the Moore-Penrose pseudoinverse with a suitable regularization. In this talk, I will take the nonlinearity of the inverse problem into account by considering the inversion of the Born and Rytov series. Since the Rytov approximation is more practical and has been used for experiment and clinical research, in particular I will explore nonlinear Rytov approximations by constructing the inverse Rytov series.

Inferring properties of the human brain from clinical and experimental data

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The human brain is one of the most complicated systems known to humankind. With around 85 billion neurons and nearly 100,000 miles of blood vessels, it is a highly complex, self-organised, active system. However, measuring brain structure and function is surprisingly difficult. Despite major advances in imaging technologies, our understanding of both the structure and the function of the brain remains surprisingly poor. Mathematical models thus play an important role in interpreting clinical and experimental data. I will present work in my group that has led to the development of multiple scale methods to simulate cerebral blood flow, oxygen transport, and water movement. These have now been used to simulate these processes in whole-brain models for the first time within a mathematically rigorous, yet computationally inexpensive framework.

A key difficulty with these models, however, remains the issue of parameter estimation. I will present our preliminary work on parameter estimation and the use of optimisation techniques, together with data from a potential new imaging modality. However, much work remains to be done to obtain robust estimates of model parameters across different length scales from both experimental data (both human and animal) and clinical data. Potential future avenues for this will be discussed with a view to drawing on the expertise of others at the workshop.

Control of discrete-time stochastic systems and application to remote automated driving

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This talk is mainly about control theory for discrete-time systems whose state transition is determined randomly. In the linear case, this kind of stochastic systems can be described by state equations with coefficient matrices given by discrete-time matrix-valued stochastic processes. In this talk, recent results on second-moment stability of the stochastic systems and associated Lyapunov inequalities are first introduced, without restricting the class of the associated stochastic processes. Then, some theoretical extensions are further discussed. As one of the possible practical applications of such stochastic control theory, results obtained in university-industry collaborative research on remote automated driving are also introduced.

Globally injective and bijective neural operators

Takashi FURUYA (Shimane University)*

Recently, there has been great interest in neural operators [2], which learn operators between infinite-dimensional function spaces. Given their discretization-invariance property, neural operators have been remarkably successful, especially in learning solution operators for partial differential equations (PDEs). In this talk, we show that injective neural operators are universal approximators of continuous operators. This holds substantial relevance to inverse problems in PDEs, as injective neural operators can serve invertible surrogate models for solution operators, which, in general, is not injective due to ill-posnedss of inverse problems. This talk bases on [1], which is a joint work with Michael Puthawala (South Dakota State University), Matti Lassas (University of Helsinki), and Maarten V. de Hoop (Rice University).

References

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Stability for nonlinear inverse problems with low dimensional priors

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In this talk I will review some stability properties of a large class of inverse problems in infinite dimensional (Banach) spaces, especially nonlinear and ill-posed (e.g. electrical impedance tomography). The main focus will be on how to improve stability by imposing low dimensional priors on the unknown, in a deterministic setting, using functional analytic techniques. The low dimensional priors considered will be mainly finite dimensional linear subspaces and finite dimensional manifolds. These results can be combined with manifold learning techniques (in particular based on generative models) to learn a low dimensional parameterization of the unknown, which yields again stability under suitable conditions. This is based on a series of works done in collaboration with G.S. Alberti (University of Genoa), A. Arroyo (Complutense Madrid), J. Hertrich (TU Berlin), S. Sciutto (University of Genoa).

Inverse problems of nonlinear wave equations

Xi Chen

Shanghai Centre for Mathematical Sciences, Fudan University

We shall give a survey of some nonlinear wave equations and inverse problems arising in mathematical physics. In particular, we shall present recent progress on stable recovery of coefficients of nonlinear wave equations.

Bayesian inversion in a trans-dimensional framework for subsurface stratification

Michael Conrad Koch

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Abstract

Subsurface stratification from Cone Penetration Test (CPT) data to delineate different soil layers and their layer boundaries is an important task in soil investigation exercises in agricultural, geotechnical and geological engineering. The stratification parameters are: the number of soil layers, the layer interface depths and the soil spatial random field parameters. Seeing that the number of layers is unknown apriori and that the other two parameters are dependent on the number of layers, the problem assumes a trans-dimensional characteristic where the dimension of the parameter space can change. Using a Bayesian formulation, a simple proposal density is designed that satisfies the reversibility criteria in the Reversible Jump Markov Chain Monte Carlo (RJMCMC) framework. The proposal density enables jumping between parameter spaces of different dimensions and forms part of the first sampling block in a three-block MCMC algorithm. The other two blocks involve sampling of the spatial random field parameters. RJMCMC algorithms typically display poor acceptance rates necessitating the development of efficient samplers. The particular blocking strategy described allows for the development of a formulation that primarily involves computationally inexpensive tasks such as sampling from truncated normal and Inv-Gamma distributions and evaluation of general normal densities. The algorithm is validated using synthetic and real world CPT data. A standard normality check of the decorrelated residuals is seen as a useful measure to test algorithm performance.

Identification of Conductivity in Elliptic equations using Deep Neural Networks

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The focus of this talk is on the numerical methods used to identify the conductivity in an elliptic equation. Commonly, a regularized formulation consists of a data fidelity and a regularizer is employed, and then it is discretized using finite difference method, finite element methods or deep neural networks. One key issue is to establish a priori error estimates for the recovered conductivity distribution. In this talk, we discuss our recent findings on using deep neural networks for this class of problems, by effectively utilizing relevant stability estimates.

The field concentration problem in nano-optics

Sanghyeon Yu

Department of Mathematics, Korea University

Composite materials shows the high field concentration when the inclusions get close to touching. This phenomenon has many practical applications in imaging, spectroscopy, and meta-materials. In this talk, we discuss a new way of tackling the field concentration problem via the spectral analysis of the Neumann-Poincare operator. We shall also discuss a related inverse problem.

Modeling of slip on a fault during an earthquake: point-source approximation

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Some parts of the Earth's crust dynamically rupture and slip during earthquakes, and their deformation is often represented by the equation of motion of an elastic domain $\Omega(\ni \boldsymbol{x})$:

$$\rho\, \ddot{\boldsymbol{u}} = \nabla \cdot \left(\lambda \left(\nabla \cdot \boldsymbol{u}\right) \boldsymbol{I} + \mu \left(\nabla \boldsymbol{u} + \left(\nabla \boldsymbol{u}\right)^T\right)\right) \tag{1}$$

where $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t)$ is the displacement vector, and $\rho = \rho(\boldsymbol{x})$, $\lambda = \lambda(\boldsymbol{x})$, and $\mu = \mu(\boldsymbol{x})$ are the density and Lamé parameters in the heterogeneous Earth, respectively. While the domain Ω is a continuum, the velocity during the coseismic slip must be discontinuous across the ruptured region $\Gamma(\subset \Omega)$, called a fault. Seismologists focus on the gap of displacement and velocity across Γ. Here, we assume that Γ is planar and lying along x_1-x_2-plane, and the gap is parallel to x_1-axis, for simplicity. Then, the slip at $\boldsymbol{s}(\in \Gamma)$ is defined as:

$$D(\boldsymbol{s}, t) := \lim_{\varepsilon \downarrow 0} \left[u_1(\boldsymbol{s} + \varepsilon \boldsymbol{\nu}, t) - u_1(\boldsymbol{s} - \varepsilon \boldsymbol{\nu}, t)\right], \tag{2}$$

where $\boldsymbol{\nu} = (0, 0, 1)^T$ is a normal vector to Γ.

The forward and inverse problems to model slip D and slip velocity $\dot{D}$ are a major topic in seismology. When we observe ground velocity $\dot{\boldsymbol{u}}$ at sufficiently far field, the spatial extent of Γ would be negligible, and $\dot{\boldsymbol{u}}$ is synthetically represented as follows:

$$\dot{\boldsymbol{u}}(\boldsymbol{x}, t) = \int_0^t \dot{D}(\tau)\, G(\boldsymbol{x}, t - \tau; \boldsymbol{s})\, d\tau, \tag{3}$$

where G is a propagator obtained synthetically or empirically. Some empirical laws are known for the moment and spectra of $\dot{D}$ in this point-source approximation. The author will discuss their main points and a stochastic model to represent the empirical laws.

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Scattering analysis of guided wave beam by defects in a plate with finite width

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Guided wave scattering arises in nondestructive testing (NDT) using ultrasonic guided waves. The guided waves display dispersion and multimodal characteristics, making it crucial to employ dispersion analysis to understand them [1]. Additionally, scattering analysis necessary to comprehend the received waves in NDT. Typically, numerical methods are used to conduct dispersion and scattering analyses. This study proposes a hybrid semi-analytical finite element-boundary element method (SAFE-BEM) for this purpose.

The dispersion curves for a three-dimensional plate with finite width are complex, making it challenging to directly employ them for the analysis of incident and received waves in experimental measurement. In conventional guided wave measurement, the incident and received waves are considered using two-dimensional dispersion curves for an infinite-width plate [2]. To understand the incident and received waves in terms of the two-dimensional dispersion properties, we examine the appropriate incident wave and scattered far-fields in numerical simulations. For this purpose, the Gaussian beam of plate wave [3] is considered as an incident wave.

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Dual-grid parameter choice method for total variation regularized image deblurring

Markus Juvonen

Department of Mathematics and Statistics, University of Helsinki

I will talk about a new parameter choice method for total variation (TV) deblurring of images which is based on a dual-grid computation of the solution. Instead of a single grid we have two grids with different discretisation. The first grid is the same were the measurement is given. The origin of the second grid is shifted half a pixel width both horizontally and vertically. Note that the underlying true image is the same for both grids. Assume that the pixel size is much smaller than a typical constant valued area in an image. The premise of the study is that when solving the TV regularised noisy deblurring problem with a large enough parameter the solutions on both grids will converge to the same image. The proposed algorithm looks for the smallest parameter with which convergence can be numerically detected. The method has been tested on both simulated and real image data. Computational experiments suggest that an optimal parameter can be chosen by monitoring the relative difference of the TV seminorms of the dual-grid solutions while changing the regularisation parameter size. This talk is based on collaboration with Yiqiu Dong (DTU), Samuli Siltanen (University of Helsinki), Matti Lassas (University of Helsinki) and Ilmari Pohjola (University of Helsinki).

Electrical impedance tomography and virtual X-rays

Samuli, Siltanen

Industrial Mathematics, University of Helsinki

A connection between Electrical Impedance Tomography (EIT) and X-ray tomography was found in [Greenleaf et al. 2018] using microlocal analysis. Fourier transform applied to the spectral parameter of Complex Geometric Optics solutions produces virtual X-ray projections, enabling a novel filtered back-projection type nonlinear reconstruction algorithm for EIT. This approach is called Virtual Hybrid Edge Detection. It is remarkable how this new approach decomposes the EIT image reconstruction process in several steps, where all ill-posedness is confined in two linear steps. Therefore, we can separate the nonlinearity and ill-posedness of the fundamental EIT problem.

Approximate peak time and its application to time-domain fluorescence diffuse optical tomography

Shuli Chen

Department of Mathematics, Hokkaido University

This talk will present an inverse problem for the fluorescence diffuse optical tomography (FDOT) identifying multiple point targets. The targets are the fluorescent agents. The FDOT process is modeled by two diffusion equations coupled with the source term. Our measured data are the time-domain data, measured at some detection points as the temporal response of the fluorescence intensity to an instantaneous injection of the excitation light from a source point. The peak time, clearly observed in the space-time data, has been used as an index to detect the targets. We introduce an approximate peak time based on an asymptotic analysis, which agrees very well with the peak time obtained by the numerical calculation using typical optical parameters. Then, using approximate peak time and combining it with the bisection method, we propose a mathematically rigorous inversion method for the FDOT. The proposed method is efficient and robust and accurate for identifying target locations embedded in deep depth. This is a joint work with Junyong Eom, Gen Nakamura and Goro Nishimura.

A Mathematical Theory of Computational Resolution Limit

Hai Zhang

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It is well-known that the resolution of optical imaging system is fundamentally limited by the optical wavelength. Based on this, Rayleigh proposed the Rayleigh criterion on the minimum resolvable distance between two point sources, the so called Rayleigh limit. Although widely used in the practice, this limit is not so useful for images that are subject to elaborated data processing. To remedy this, we develop a theory of computational resolution limit to characterize the fundamental resolution limit from the approximation theory point of view. The theory can be used to explain the phase transition phenomenon in the reconstruction problem. New efficient super-resolution algorithm is also developed following the theory.

On numerical instability of a singular integral equation in x-ray computerized tomography with partial measurement

Hiroshi Fujiwara (Kyoto Univ.), Kamran Sadiq (RICAM), and Alexandru Tamasan (University of Central Florida)

We consider x-ray tomography with limited data measured only on an arc of the boundary. A Cauchy-type boundary integral formula has been proposed by Bukhgeim (1995), and the jump relation applied to it derives a Cauchy-type singular integral equation. It is well known that its iversion is unstable in the L^2 sense even if it exists. This talk presents an quantitative estimate of instability under some discretization, and it shows that the instability is not severe. Some numerical examples are exhibited to support our analysis and to show feasibility of the proposed algorithms. We note that the proposed reconstruction procedure is neither statistical nor iterative.

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